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Book Reviews

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Singular Perturbation Theory: Mathematical and Analytical Techniques with Applications to Engineering

R. S. Johnson, Springer, New York, 2005, 292 pp., $129.00

It is appropriate that the publication/copyright record for this newest book on singular perturbation theory straddles the 2004–2005 period, marking the centennial of Prandtl's 1904 presentation of his boundary-layer paper at an international mathematical congress in Heidelberg and its appearance in the proceedings of the congress the following year. It would be hard to overestimate the significance of the seminal insight of Prandtl and its consequences. As most readers of this journal are likely to know, most simply, singular perturbation theory is relevant to problems governed by differential equations containing a small parameter. To obtain an approximate, if possible analytic, solution, a straightforward expansion in the parameter, based on its smallness, is carried out (an approach much exploited by Poincaré in his work in celestial mechanics). If the resulting solution is accurate throughout the entire spatial (and temporal, if unsteady) domain(s), this solution is referred to as being "uniformly valid," and the problem is called a regular perturbation problem. If this is not the case, the straightforward expansion is not uniformly valid and the problem is said to be a singular perturbation problem and must be treated with special methods. The most widely used of these methods is the method of matched asymptotic expansions, wherein, most simply, the straightforward asymptotic expansion is retained but supplemented by an asymptotic expansion in rescaled independent variables, rescaled to "capture" in the region of nonuniformity the essential physics of the problem not captured by the straightforward expansion, the asymptotic matching of these two expansions yielding a uniformly valid approximation. The 1904 paper of Prandtl had all of these essentials of matched asymptotic expansions, the straightforward expansion, or outer expansion, being the inviscid flow "far" from the body, the inner expansion near the body surface being the viscous boundary layer, and the asymptotic matching conditions being the familiar ones of boundary-layer theory. It was 50 or so years before researchers, prominently at Caltech, for high Reynolds numbers (Kaplun, Lagerstrom, et al.), and at Cambridge University, for low Reynolds numbers (Proudman, Pearson, et al.), provided the formal structure for Prandtl's great insight and construction. The next 50 years saw enormously important applications of the method broadly across the physical sciences and engineering, improvements in the method, and the development of other techniques for obtaining uniformly valid approximations for singular perturbation problems. The latter includes, importantly, the method of strained coordinates and the method of multiple scales.

This text is the second in a new series by Springer, the series title being exactly the same as the subtitle of this book. It is based on material taught by the author, a distinguished applied mathematician at the University of Newcastle, and is very much a textbook, filled with a very large number of worked examples and end-of-chapter exercises, with answers and hints for the latter in the back of the book. These are generally well chosen and provide evidence of the three decades the author has taught this material. Appropriate references are cited throughout the book. The author writes in an appealing conversational style, without shying away from the necessary mathematical details, which should help to alleviate much of the foreboding such material typically elicits from students.

A sense of the emphasis on applications is evident in the titles and content of the chapters that comprise the book. Chapter 1, Mathematical Preliminaries, 41 pages, provides in an economically few pages the necessary definitions and concepts of asymptotic expansions, issues of convergence/divergence, uniformity, overlap regions and intermediate variables, the matching principles, and composite expansions. This is followed by the second chapter, Introductory Applications, which applies these ideas mainly to ordinary differential equations and introduces the concept of a boundary layer or transition layer. The third chapter continues with further applications, mainly involving partial differential equations. In this chapter the PLK method, often nowadays called the method of strained coordinates, is presented. The method of multiple scales, arguably the newest, most difficult, and most powerful of methods to treat singular perturbation problems, comprises the fourth chapter. Treated in this chapter are turning-point problems and the WKB technique for ordinary differential equations, concluding with applications to partial differential equations, in the context of wave propagation, which include a discussion of dispersive and nondispersive waves, the nonlinear interaction of dispersive waves, and the nonlinear Schrödinger equation. The concluding and longest chapter of the book, Chapter 5, applies the methods developed in the earlier chapters to a selection of problems from various scientific fields, grouped into mechanical and electrical systems, celestial mechanics, physics of particles and light, semi- and superconductors, fluid mechanics, extreme thermal processes, and chemical and biochemical reactions.

There is a distinguished history of excellent texts and monographs in singular perturbation theory and applications, with various degrees of emphasis on the mathematics vs applications. In this history, spanning four decades, are the books by van Dyke, Kaplun, Lagerstrom, Kevorkian and Cole, Hinch, Nayfeh, and O'Malley, to name only some of the best-known ones. This newest addition to the list is a very readable volume, with a fine balance between theory and applications, the latter extending far beyond fluid mechanics and aerospace. It would be an excellent text for an applied mathematics course on singular perturbations. Its accessibility and clarity would make it equally valuable to someone not familiar with the subject who wished, through self-study, to do so. Finally, even those well-versed in the subject are likely to gain by filling in gaps, or seeing interesting new applications of the century-old revolutionary ideas introduced by Ludwig Prandtl.

Stanley A. Berger
University of California, Berkeley